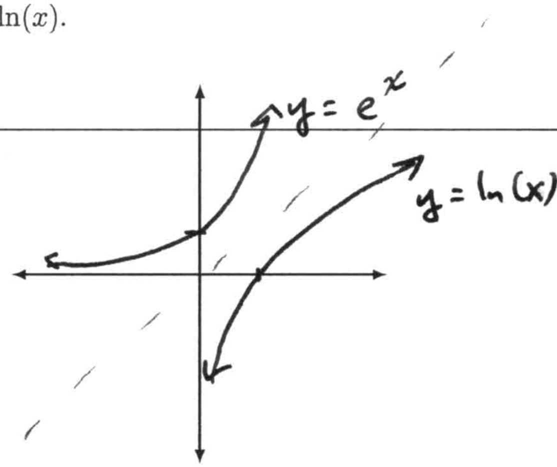
Inverses Review Worksheet | Math 1060 | Class #27

Name: Solutions | Section: ____________

Logarithmic Functions

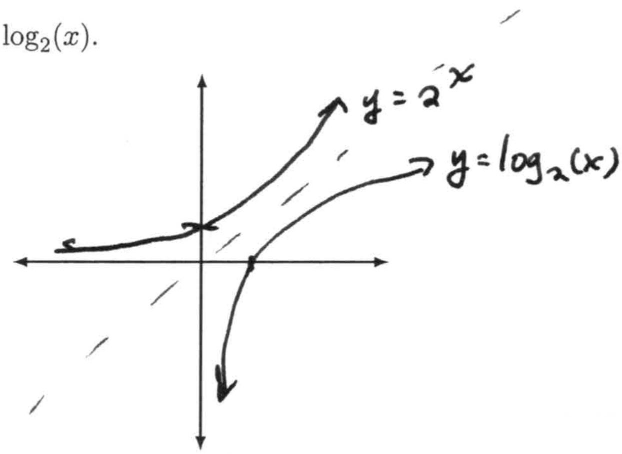
1. Graph $y = e^x$ and $y = \ln(x)$.

$\ln(x)$ is the inverse of e^x



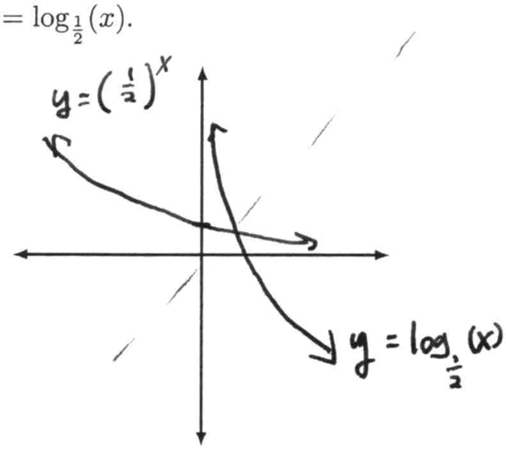
2. Sketch $y = 2^x$ and $y = \log_2(x)$.

$\log_2(x)$ is the inverse of 2^x



3. Sketch $y = \left(\frac{1}{2}\right)^x$ and $y = \log_{\frac{1}{2}}(x)$.

$\log_{\frac{1}{2}}(x)$ is the inverse of $\left(\frac{1}{2}\right)^x$



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4. Simplify the following expressions:
   (a) $\log_2(16) = \log_2(2^4) = 4$
   (b) $\log_4(16) = \log_4(4^2) = 2$
   (c) $\log_{16}(16) = \log_{16}(16^1) = 1$

5. Simplify the following expression:

$$e^{5\ln(2)}$$

$$= \left(e^{\ln(2)}\right)^5$$

$$= 2^5$$

6. Simplify the following expression:

$$\ln(e^{5x})$$

$$= 5x$$

7. Simplify the following expression:

$$10e^{\left(\frac{\ln(2)}{3}t\right)}$$

$$= 10 \cdot e^{\ln(2) \cdot \frac{t}{3}}$$

$$= 10 \cdot \left(e^{\ln(2)}\right)^{\frac{t}{3}}$$

$$= 10 \cdot (2)^{t/3}$$

8. Simplify and compare the expressions $e^{\frac{1}{2}\ln(x)}$ and $e^{x\ln(\frac{1}{2})}$.

$$e^{\frac{1}{2}\ln(x)} = \left(e^{\ln(x)}\right)^{\frac{1}{2}} = x^{\frac{1}{2}} = \sqrt{x}$$

$$e^{x\ln(\frac{1}{2})} = \left(e^{\ln(\frac{1}{2})}\right)^x = \left(\frac{1}{2}\right)^x$$

these are very different!

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9. Rewrite the following expression as a single logarithm:

$$\left(\ln(x+5) - \ln(x-2)\right) + \ln(x)$$

$$= \ln\left(\frac{x+5}{x-2}\right) + \ln(x)$$

$$= \ln\left(\left(\frac{x+5}{x-2}\right) \cdot x\right)$$

$$= \ln\left(\frac{x^2+5x}{x-2}\right)$$

10. Rewrite the following expression as a single logarithm:

$$2\ln(x+1) + 3\ln(x+2)$$

$$= \ln\left((x+1)^2\right) + \ln\left((x+2)^3\right)$$

$$= \ln\left((x+1)^2(x+2)^3\right)$$

11. Rewrite the following expression as a single logarithm:

$$\ln(x+3) - 2\ln(x) + \ln(x+1)$$

$$= \left(\ln(x+3) - \ln(x^2)\right) + \ln(x+1)$$

$$= \ln\left(\frac{x+3}{x^2}\right) + \ln(x+1)$$

$$= \ln\left(\frac{(x+3)(x+1)}{x^2}\right)$$

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Exponential Growth and Decay

1. What is the basic formula for exponential growth and decay?
   What is the physical meaning of each constant?

$$P(t) = P_0 \cdot e^{kt}$$

P_0 = starting population

k = growth constant

2. What is the difference in the constants that occur in growth and in decay?
   Is $\ln(2)$ positive or negative? What about $\ln\left(\frac{1}{2}\right)$?

k is positive in exponential growth,

and negative in exponential decay

$\ln(2)$ is positive

$\ln\left(\frac{1}{2}\right) = \ln(2^{-1}) = -1 \cdot \ln(2)$ is negative

3. What does the term "half life" mean?

The half life of a radioactive substance is the time you need to wait to get the amount of material cut in half.

Eg: if you start with 10 grams & you have a half life of 7 years

$$P(7) = 5$$

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4. Suppose that a population of bacteria is growing in a petri dish. Suppose also that the first time you look at the dish you count 20 bacteria, and that you count 200 bacteria in the dish 3 hours later.

   (a) Find a formula for the population as a function of the number of hours t since your first measurement.

   (b) Estimate the number of bacteria after 24 hours.

   (c) How long until the dish contains 1000 bacteria?

   (d) Find the number of hours needed for the population to double.

(a) $P(t) = P_0 \cdot e^{kt}$

$P_0 = 20$

$P(t) = 20 \cdot e^{kt}$ ← need to find k

know $P(3) = 200 = 20 \cdot e^{k \cdot 3}$

$200 = 20\, e^{k \cdot 3}$

$10 = e^{k \cdot 3}$

$\ln(10) = k \cdot 3$

$k = \frac{\ln(10)}{3}$

So $\boxed{P(t) = 20 \cdot e^{\left(\frac{\ln(10)}{3}\right)t}}$

(b) $P(24) = 20 \cdot e^{\left(\frac{\ln(10)}{3}\right)24} = 20 \cdot e^{\ln(10) \cdot \frac{24}{3}} = 20 \cdot \left(e^{\ln(10)}\right)^{\frac{24}{3}}$

$= 20 \cdot (10)^{\frac{24}{3}}$

(c) $1000 = P(t)$

$\Leftrightarrow$

$1000 = 20 \cdot e^{\left(\frac{\ln(10)}{3}t\right)}$

$50 = e^{\frac{\ln(10)}{3}t}$

$\ln(50) = \frac{\ln(10)}{3} t$

$t = \frac{3 \cdot \ln(50)}{\ln(10)}$

(d) $40 = P(t)$

$\Leftrightarrow$

$40 = 20 \cdot e^{\frac{\ln(10)}{3}t}$

$\Leftrightarrow$

$2 = e^{\left(\frac{\ln(10)}{3}t\right)}$

$\ln(2) = \frac{\ln(10)}{3} t$

$t = \frac{3 \cdot \ln(2)}{\ln(10)}$

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5. Suppose that you begin with 100 grams of a radioactive substance. Suppose also that the substance has a half life of 3 years.

   (a) Find a formula for the amount of radioactive substance remaining after t years.

   (b) What is the weight of the radioactive substance that remains after 9 years?

   (c) ~~How long must you wait for the weight of radioactive substance to drop to 10 grams?~~

(a) $P(t) = P_0 \cdot e^{kt}$

$P_0 = 100$ g.

$P(t) = 100 \cdot e^{kt}$

need to find k.

know

$P(3) = 50 = 100 \cdot e^{k \cdot 3}$

Solve for k: $50 = 100 \cdot e^{3k}$

$\frac{1}{2} = e^{3k}$

$\ln(\frac{1}{2}) = 3k$

$k = \frac{\ln(\frac{1}{2})}{3}$

$$\boxed{P(t) = 100\, e^{\frac{\ln(\frac{1}{2})}{3} t}}$$

(b) $P(\boxed{9}) = 100 \cdot e^{\frac{\ln(\frac{1}{2})}{3} \boxed{9}}$

$= 100\, e^{\ln(\frac{1}{2}) \cdot 3}$

$= 100 \cdot \left(e^{\ln(\frac{1}{2})}\right)^3$

$= 100 \cdot \left(\frac{1}{2}\right)^3$

$P(9) = \frac{100}{8}$

(c) find t s.t.

$P(t) = 10.$

$10 = 100 \cdot e^{\frac{\ln(\frac{1}{2})}{3} t}$

$\frac{1}{10} = e^{\frac{\ln(\frac{1}{2})}{3} t}$

$\ln(\frac{1}{10}) = \frac{\ln(\frac{1}{2})}{3} t$

$\frac{3 \cdot \ln(\frac{1}{10})}{\ln(\frac{1}{2})} = t$

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General Inverses

1. Let $f(x) = x^3 - 1$. Find an equation for $f^{-1}(x)$.

$$y = x^3 - 1$$

$$y + 1 = x^3$$

$$\sqrt[3]{y+1} = x = f^{-1}(y)$$

So

$$\boxed{f^{-1}(x) = \sqrt[3]{x+1}}$$

2. Let $f(1) = 5$, $f(5) = 6$, $f(6) = 9$, and $f(9) = 1$. Find the following:

(a) $\boxed{f^{-1}(9) = 6}$

$f^{-1}(9) = x \Leftrightarrow 9 = f(x) \Rightarrow x = 6$

(b) $\boxed{f^{-1}(6) = 5}$

$f^{-1}(6) = x \Leftrightarrow 6 = f(x) \Rightarrow x = 5$

(c) $\boxed{f^{-1}(f^{-1}(6)) = 1}$

(from before $f^{-1}(6) = 5$)

$= f^{-1}(5)$

$= 1$

(d) $\boxed{f^{-1}(f^{-1}(9)) = 5}$

(from before $f^{-1}(9) = 6$)

$= f^{-1}(6)$

(from before $f^{-1}(6) = 5$)

$= 5$

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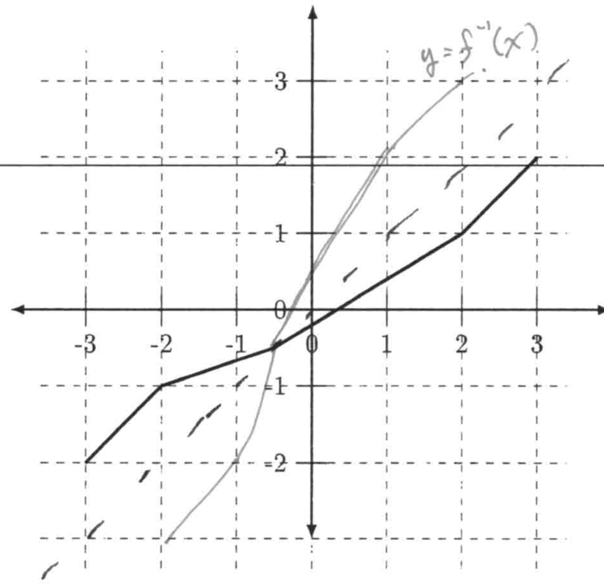
3. Suppose $f(x)$ is defined using the graph below.



(a) Does $f(x)$ have an inverse? Justify (explain) your answer.

Yes - it is one-to-one

- it passes the Horizontal line test

- each output comes from a SINGLE input

(b) Find $f^{-1}(2)$ and $f^{-1}(-2)$.

$f^{-1}(2) = x$ $\Leftrightarrow$ $2 = f(x)$ $\Leftrightarrow$ $x = 3$ $\boxed{f^{-1}(2) = 3}$

$f^{-1}(-2) = x$ $\Leftrightarrow$ $-2 = f(x)$ $\Leftrightarrow$ $x = -3$ $\boxed{f^{-1}(-2) = -3}$

(c) Sketch the graph of $f^{-1}(x)$ on the axes above.

See above!

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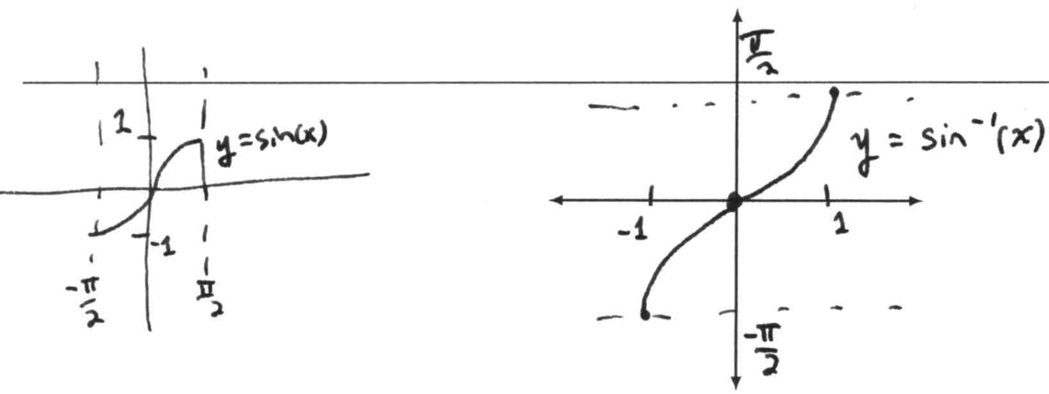
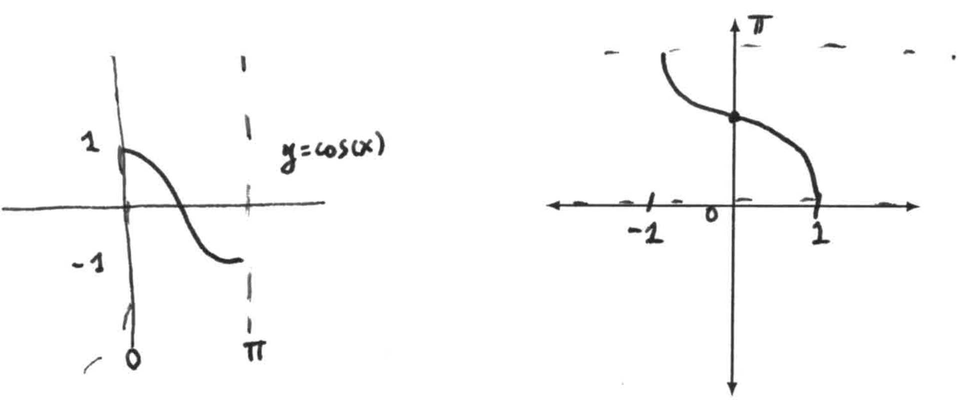
1 Trigonometric Inverses

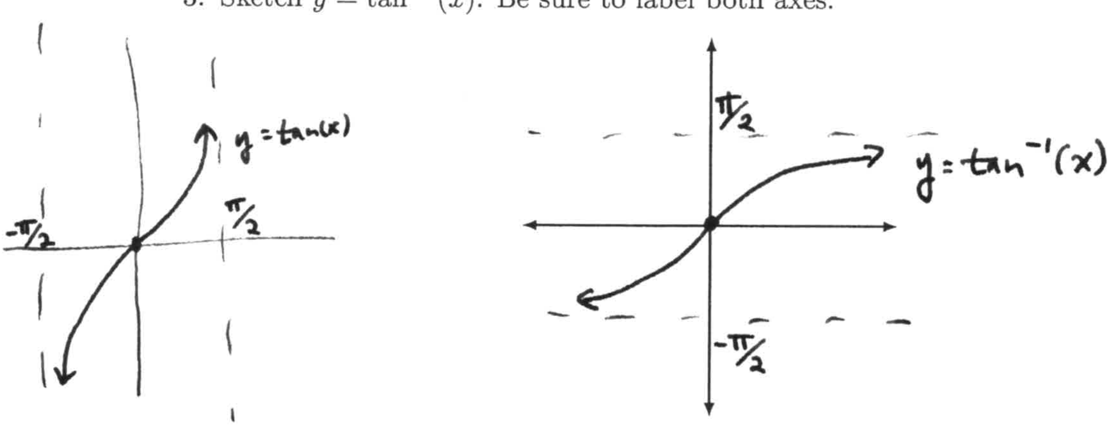
1. Sketch $y = \sin^{-1}(x)$. Be sure to label both axes.



2. Sketch $y = \cos^{-1}(x)$. Be sure to label both axes.



3. Sketch $y = \tan^{-1}(x)$. Be sure to label both axes.



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4. Compute $\sin^{-1}\left(\frac{-1}{2}\right)$.

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5. Compute $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

6. Compute $\cos^{-1}\left(\frac{-1}{2}\right)$.

7. Compute $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

8. Compute $\tan^{-1}(-1)$.

9. Compute $\tan^{-1}(\sqrt{3})$.